End Term Examination
Classical Mechanics

B. Math., 2^{nd} Year, July - November 2024,
Date and Time: November 18, 2024, 10 a.m,
Duration: 3 hours,
Total points: 80

You must provide clear argument(s) for all steps for full grade. An answer with zero explanation, even if correct, will not receive any credit.

1. A block of mass m is initially held motionless on a frictionless inclined plane of incline θ. The inclined plane (whose mass is $M(> m)$) itself rests on a frictionless horizontal surface. The block is now released. Find the acceleration of the inclined plane.

   10 points

2. A vertically upright circular hoop of radius R rotates about the vertical diameter with an angular frequency ω. A bead of mass m is free to slide along the hoop without friction. Let $\theta(t)$ be the angle of the bead sustained at the center of the hoop as measured from the downward vertical axis.

   (a) Find the equation of motion of the generalized coordinate θ.

   (b) Find the points of equilibrium of the bead.

   (c) Find the frequency of small oscillations about the stable equilibrium point(s).

   7 + 5 + 8 = 20 points

3. Imagine a particle of mass m moving in a central potential of the form $-\alpha/r^2$, $\alpha > 0$. The value of the angular momentum is fixed at L. In an $E - r$ diagram, analyze the motion for the various values of total energy E and α qualitatively.

   15 points

4. Consider the case of the double pendulum - the top pendulum is a massless rod of length l_1 with a point mass m_1 affixed at the bottom; the pendulum itself is fixed immovably at the top. The bottom pendulum is affixed to m_1 at the top, and is itself a massless rod of length l_2, with a point mass

m_2 at the bottom. The double pendulum moves in a vertical plane, and the customary general co-ordinates are the angles the rods make with the downward vertical.

(a) Using the co-ordinates as above, derive the equations of motion of the double pendulum.

(b) Make a small-angle approximation up to linear order and find the approximate equations of motion.

(c) Use $l_1 = l_2 = l$, along with the small angle approximation. Find the normal frequencies of oscillation.

(d) Discuss the motion in the following cases: $m_1 \gg m_2$ and $m_1 \ll m_2$ (Hint: for $m_1 \gg m_2$, write $m_2/m_1 = \epsilon \ll 1$ and expand the frequncies up to leading order in terms of ϵ. Similarly for the other limit.)

7 + 5 + 5 + 8 = 25 points

5. Imagine a horizontal disc of radius R rotating in a counter clockwise direction with constant angular velocity ω. Imagine there is a radial line painted on the disc from the center to the rim. A person, initially standing on the rim edge of the line walks towards the center of the disc, with a radial speed v in the rotating frame, all the while being on the line.

(a) To an inertial observer, what are the horizontal forces acting on the person? Specify their directions as well.

(b) To an observer attached to the disc, find the horizontal forces acting on the person along with their magnitude and direction.

(c) Find $d\vec{L}/dt$ in the inertial (lab) frame; $\vec{L}$ being measured from the center of the disc. Which external force is responsible for a non-zero $d\vec{L}/dt$, if any?

3 + 3 + 4 = 10 points